MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ECONOMICS
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02142-1347

12 May 2008

Professor William Ziemba
University of British Columbia
Sauder School of Business
2053 Main Mall
Vancouver, BC
Canada V6T 1Z2

Dear Bill,

I marvel that you still cherish hankerings for the Kelly-Breiman-Latané criterion.

I enclose a 2005 letter from me to Elwyn Berlekamp of Berkeley, hoping to set him straight on that topic. Also, I have marked up by ballpoint pen your one-page email letter to me.

I don't perceive that in 1986 you made any case for persons who don't have logarithmic utility to respect the injunction that they should act as if they did have it. My 2005 letter spells out the metric harm that possessors of $\sqrt{W}$ or A-1/W utilities would do to themselves if they listened to Kelly or Jim Thorpe or … .

Best,

Paul

Paul A. Samuelson

Encs. (2)

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16 November 2005

Professor Elwyn Berlekamp
2039 Shattuck Avenue, #408
Berkeley, CA 94704

Dear Elwyn,

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Here is a fable that might be of some interest. I have three "rational" neighbors: Tom, Dick and Harriet. All are risk averters who shun all "unfair" gambles. Being of unequal degree of risk aversion, Harriet is the most cautious; Dick (Goldilocks) has more risk tolerance than Harriet but less risk tolerance than sportive Tom. All face one serious problem. They must invest for a future period of retirement while the feasible choice is between (1) safe cash with only *zero* yield, and (2) one single stock that in *every* period will, for each $1 invested in it now, bring to the investor one period from now either \$4 or $\$\frac{1}{4}$ at *even* odds.

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Q. Given a horizon of $N > 1$ periods until the final date of your retirement, will you change your x proportions? To the surprise of lay people, all three answer No, no change.

Test II. Now you're given the option of *blending* cash and stock. Harriet replies: for every N, large or small, I'll put $\frac{2}{9}^* = x^*$ in the stock and $\frac{7}{9}^* = 1-x^*$ in safe cash. No surprise when Tom reports a larger x^* than Harriet: for Tom, $x^* = 1^*$ and $1\text{-}x^* = 0^*$. Middling Dick, as expected, has his x^* between those of the other two. For Dick, $x^* = \frac{1}{2}^* = 1\text{-}x^*$. (Maybe Dick's last name is Kelly or Breiman or Latané.)

Why these particular decisions? Interrogations reveal that all three are devout Laplacians, who ever strive toward maximal Expected Concave Utility of wealth outcomes.

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Mr. Elwyn Berlekamp
Page 2
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$$\operatorname*{Max}_{x}\left\{\frac{1}{2}U(4x+1-x)+\frac{1}{2}U\left(\frac{1}{4}x+1-x\right)\right\}, \text{ in Test II.} \tag{1.2}$$

Dick reports his $U(W) = \log W$ (what economists call 1738 D. Bernoulli utility). What's the same thing, Dick is a Geometric Mean maximizer.

Cautious Harriet is a Harmonic Mean maximizer, with $U = -W^{-1}+A$. Her limited degree of risk tolerance fits pretty well lots of empirical Wall Street "equity premia" data.

To measure how much each person gains or loses in (subjective) "Certainty Equivalent Dollars," I define their three different CE's by the following general formulas:

$$U(CE) = \frac{1}{2}U(4)+\frac{1}{2}U\left(\frac{1}{4}\right) \tag{2.1}$$

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$$CE = GM = \sqrt{4\cdot\frac{1}{4}} = 1, \text{ for Dick} \tag{2.3}$$

$$CE = HM = \left[\frac{1}{2}\left(\frac{1}{4}\right)+\frac{1}{2}(4)\right]^{-1} = 1-\frac{9}{17}, \text{ for Harriet} \tag{2.4}$$

$$CE = KM = \left[\frac{1}{2}\sqrt{4}+\frac{1}{2}\sqrt{\frac{1}{4}}\right]^{2} = 1+\frac{9}{16}, \text{ for Tom} \tag{2.5}$$

The $\sqrt{W}$ formula of 1728 Kramer defines for Tom's CE a "root-squared Mean", KM.

$$HM < GM < KM \leq AM = \frac{1}{2}(4)+\frac{1}{2}\left(\frac{1}{4}\right). \tag{2.6}$$

Dick is a pushy guy. What if he persuades Harriet to replace her $x^* = \frac{2}{9}$ by his $x^* = \frac{1}{2}$? If she agrees to shoot herself in her own foot, the loss in her CE dollars below her best CE^* dollars is equivalent to her having agreed to throw away a definable percentage of her initial wealth. What's left, invested her *proper* way, will fall short of what she could have got by "being true to her self" by a measurable deadweight loss.

Persuasive Dick could also do a measurable \$ harm to Tom if Tom gives up $x^* = 1$ and goes along with Dick's $x^* = \frac{1}{2}$.

Bill, calculate how much Tom loses
by aping Dick Kelly for horizons of 1, 2, ..., N, ... as N grows
Do same for Harriet.

Can these one-period harms erode away after Tom and Harriet come to shoot themselves in their respective legs two times, three times, $N = 10^{10}$ times? No. No such Limit Theorem is valid. For N large, $N >> 1$, $x^* = \frac{2}{9}$ and $x^* = \frac{1}{2}$ and $x^* = 1$ each produce *on retirement date* three different wide-spread Log Normal limit distributions. Tom's Log Normal has the largest absolute arithmetic mean dollars. Harriet's has the least absolute arithmetic mean of dollars. However, at Harriet's request we calculate the three H.M.'s. Hers is the largest!

Theorem: In no run, however long, does Kelly's Rule effectuate a "dominating" retirement nest egg.

In a vanity duel between any two neighbors, where what's to be maximized is A's probability of being ahead of B when they both retire at the same time and start to invest at the same time, Dick types will beat out both Harriet types and Tom types. And for Methusala-ish neighbors, Dick's probability edge will go to 1 ("almost"), as $N \to \infty$.

Elwyn, if I am wacky, you can set me straight. If I am right, that shouldn't invalidate any important post-Shannon information-theoretic theorems.

Sincerely,

Paul

Paul A. Samuelson

P.S. For two outcome stocks, solve for x^* as the root of the first derivative equation $\frac{d}{dx}\left\{\frac{1}{2}U(3x+1)+\frac{1}{2}U\left(1-\frac{3}{4}x\right)\right\} = 0$. x^* can be found for these three neighbors by solving only a deducible *linear* equation. That's only because all three are species of the Club of Laplacians with Constant *Relative* Risk Aversion, as defined by each having $WU''(W)/U'(W)$ be a negative constant.

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13 December 2000

Professor William T. Ziemba
University of British Columbia
Faculty of Commerce
Vancouver, B.C. V6T 1ZZ
Canada

Dear William,

I cannot believe your fecundity. Two huge books in the same post. Thanks. And thanks.

You must tell me which three articles can be most profitable to me as an investor. I collect "tips," but from smart guys only.

Best,

Paul [signature]

Paul A. Samuelson

PAS/jmm

P.S. Groan. When I migrate for hibernation Saturday to Florida, I'll have to portage your two huge volumes.

Dear Paul -
This is a message and paper from Bill Ziemba. He sent to me, for you. Jim P.

Subject: Fwd: markowitz chapter
From: William Ziemba <wtzimi@mac.com>
Date: Thu, 08 May 2008 09:38:48 -0700
To: James Poterba <poterba@MIT.EDU>
CC: Rachel Ziemba <rachelziemba@mac.com>, William Ziemba <wtzimi@mac.com>

JIM would you pls print for PAS and yourself if it interests you. PAS is in this volume. THX A LOT BILL

How could a Monte Carlo simulation disprove a true theorem that Harriet and Tom are hurt by Kellyism

PAUL FYI maybe we can meet sometime to discuss the kelly views. I rather like the simulation in here that Hausch and I did in 1986. It shows for a set of independent favorable bets all with a 14% advantage over 700 trials/so medium length on time/that a large fraction of the time the investor makes huge gains. BUT its possible to make 700 independent bets all with a 14% advantage with a decent chance of winning each one at odds of 1-1 to 5-1 so probs of .19 to .57 AND LOSE 98% of ones original wealth $ 1000 in our example and fractional kelly does not help much as the min=145 or 85.5% loss.

Thorpe ~~Kelly~~ and Gross in recent press shown themselves. Jim T. did not break the bank at blackjack by using the Kelly stuff. Nor did Pimco's risk control succeed from using Kelly's U(W) = log W. Let's stay sober, PAS

THE kelly approach works well in horseracing where we can calculate exact slippage/see example in paper/and there are many many bets. HEDGE fund types like Jim Simons who do a lot of similar bets have good success too. We are finding in a trend following fund project that unceratainties in mean estimates makes it hard so far to beat 1/n ex post/research not done but 1/n is pretty formidable.

BEST wishes Bill Ziemba

PS I worked a bit on your Harry, etc example for a talk I gave at U Chicago last April. I added 2 more investors IDA for IDA MAY FULLER who was the first soc security recipient. SHe paid in 24$ and lived to be 100 and collected over 24 000 She is risk aversion approaching infinity being alpha W to the alpha for alpha going to - infinity THEN on the risky side is VICTOR for Victor Niederhoffer who is linear utility with risk aversion=0 THEN there are 5 with risk aversions infinity 2 1 1/2 and zero. VICTOR goes bankrupt for sure as VN has about 5 times/some of this about VN is discussed in a chapter in the WILEY book I did with my daughter Ziemba and Ziemba 2007 Scenarios for risk management and global investment strategies

WE are planning a kelly book reprinting the classic articles including your criticisms and the great results and commentaries SO I will draft something here to respond to your 2 letters We can then discuss that then THx for your patience on this

Begin forwarded message:

> **From:** William Ziemba <ziemba@interchange.ubc.ca>
> **Date:** May 8, 2008 8:43:58 AM PDT (CA)
> **To:** William Ziemba <wtzimi@mac.com>
> **Cc:**

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13 December 2006

Professor William T. Ziemba
University of British Columbia
Sauder School of Business
2053 Main Mall
Vancouver, BC V6T 1Z2
Canada

Dear William,

I received with pleasure a copy of your new 2006 edition of *Stochastic Optimization Models in Finance*. Many thanks.

As I hastily turned its pages, I came to an over-hasty unease that maybe you are too kind to the Kelly Criterion. (Probably I am the confused one.)

This motivates me to send you a copy of some incomplete correspondence with Elwyn Berlekamp, a Berkeley math professor who once was a Claud Shannon Ph.D. at MIT and who has made bushels of money as a trader and one-time activist in the Jim Simon Renaissance Group (Medallion Fund, etc.). He is a good friend and a colleague on the National Academy Finance Committee.

I enclose a book review by him favorable to the Kelly criterion (which I think he related to a valid Shannon theorem in modern communication theory).

I include my letter to him which used the concept of "the money certainty-equivalent to a stochastic portfolio" to measure how much of my fortune would be needlessly lost if I am a Laplacian with non-log utility (such as $\sqrt{\text{wealth}}$ or -1/wealth) and became seduced by Latané-Kelly-Breiman dogma. I deemed my arguments fatal to Kelly zealots. (However, the debate remained moot. Elwyn never answered my letter.)

Maybe I can be luckier with you?

Enjoy,

Paul

Paul A. Samuelson

PAS/jmm

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7 May 2007

Professor William Ziemba
Sauder School of Business
University of British Columbia
2053 main Hall
Vancouver
British Columbia V6T 1Z2
Canada

Dear William,

I believe I sent you a letter spelling out in exact detail how a very risk-averse Laplacian--call her Harriet--would be throwing away a computable number of her initial wealth if, instead of choosing a period-for-period Harmonic Mean (called for by *her* degree of risk aversion), she instead used *your* Kelly criterion and maximized Kelly's *Geometric* Mean.

Equally fatal would it be for Tom, a Laplacian more risk tolerant than Kelly. Tom maximizes 1728 Kramer $\sqrt{\text{Wealth Utility}}$. If investing for one period or 10^{10} periods, Tom were to use Kelly's log Utility, I calculated for you exactly how much of his initial wealth he is flushing down the toilet.

I still have been expecting your reply. Capitulation. Or cogent repudiation of my erroneous deductions. Instead you only sent me a copy of Jim Thorpe's article in your anthology. I leafed through it and threw it away. Subsequently a Nobel Prize winner sent me a copy of it, saying, "Seventy eleven times, Paul, you killed off that dragon. Yes. Thorpe made good money in Las Vegas and in Princeton. But what test is that of the log W Kelly criteria vs. the $\sqrt{W}$ Tom or the Harriet $-W^{-1}$ criteria?"

Get serious.

Amicably yours,

Paul

Paul A. Samuelson

PAS/jmm

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16 November 2005

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2039 Shattuck Avenue
Berkeley, CA 94704

Dear Elwyn,

This first of two letters thanks you for the mailing relevant to the NAS's choice among four options for paying Renaissance management fees. What the Committee must bring to your deductive diagrams is its best guess as to how the *new fund* will do in the future and how the S&P index is likely to do.

Your hunch will probably be nearer the mark than that of the rest of us because of your greater past experience with the Renaissance group. And I suspect that their past experience with you may have helped induce Jim Simon to let us in at half the usual $20 million ante. So bravo!

My other letter, drafted earlier, arises from my having been sent your valuable book review in the *American Scientist* of the recent Poundstone book. I enclose if for your possible interest, and I would benefit a lot from learning about any non-optimalities in my analyses.

Cordially,

Paul

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